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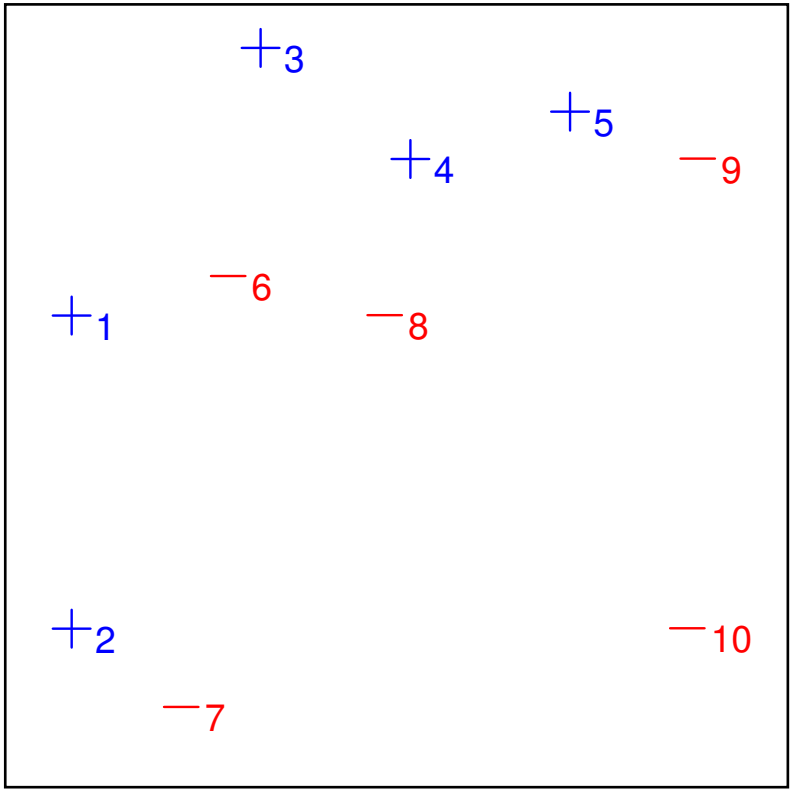
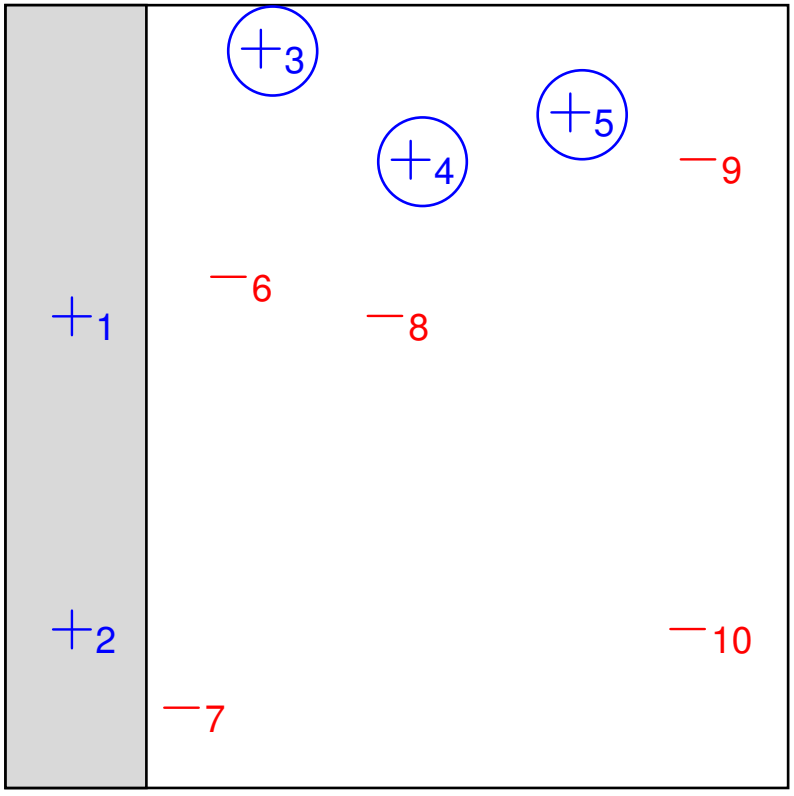
Example



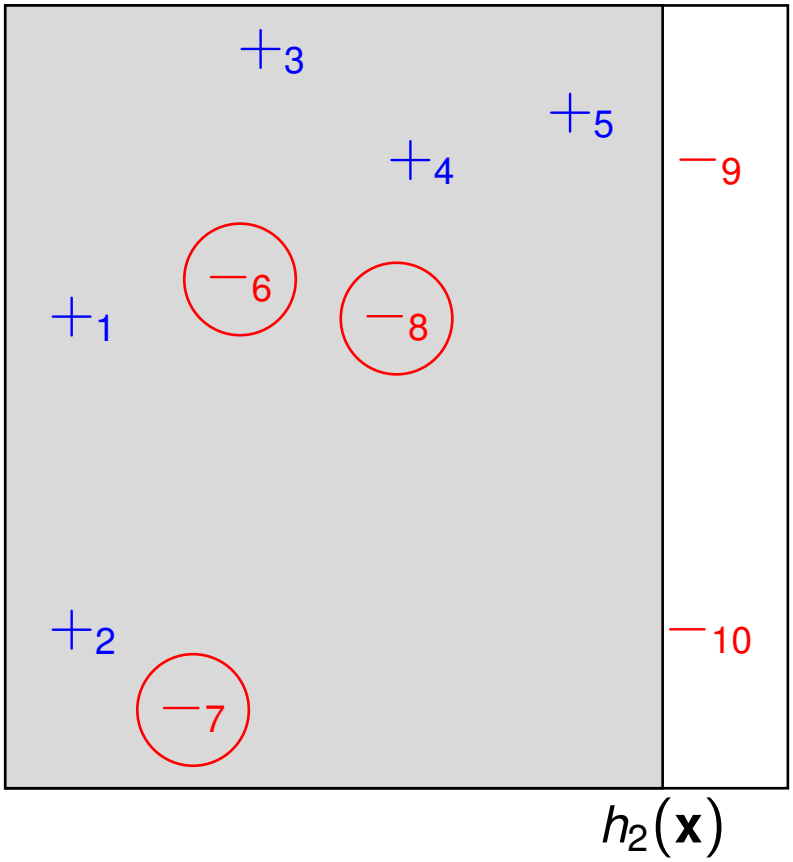
- 2 classes, $\{+, -\}$



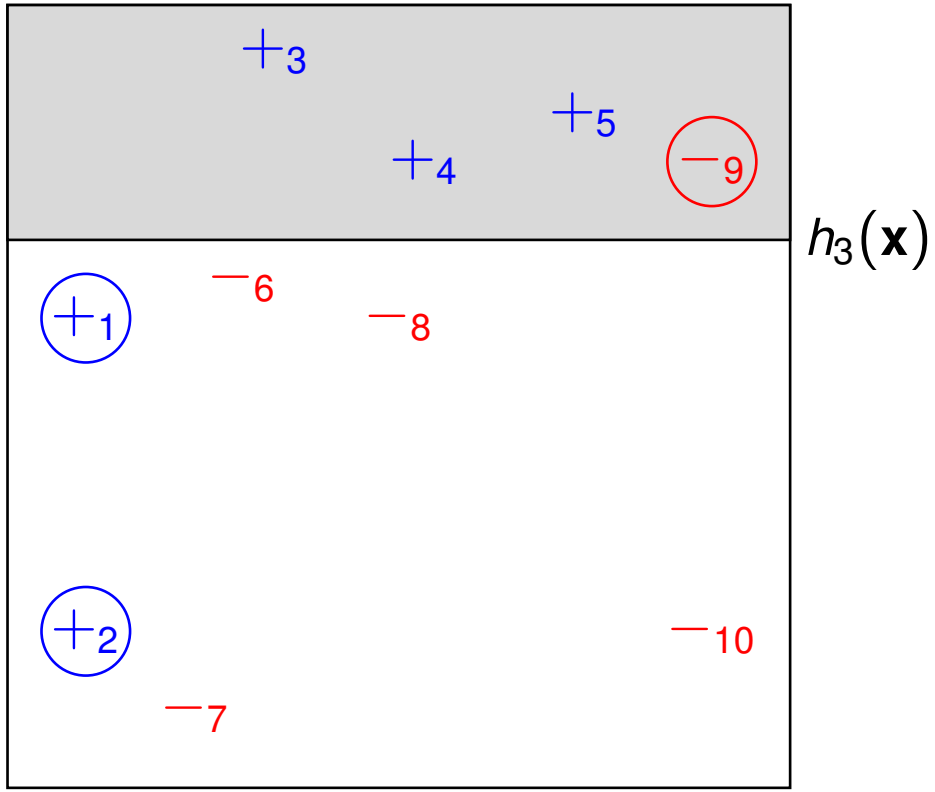
$h_1(\mathbf{x})$

- Classifier: $h_1(\mathbf{x})$
- 3 errors (3 '+' are wrongly recognised)

Example



- Classifier: $h_2(\mathbf{x})$
- 3 errors (3 '−')



- Classifier: $h_3(\mathbf{x})$
- 3 errors (2 '+' and 1 '−')

Example

Round 1: $k = 1$

- $k_{\max} = 3$ (initialisation), $\quad n = 10, \quad W_1(i) = 1/n = 0.1,\ i = 1, \ldots, 10$
- Classifier 1: $E_1 = \underbrace{0.1}_{+_3} + \underbrace{0.1}_{+_4} + \underbrace{0.1}_{+_5} = 0.3$
- Classifier 2: $E_2 = \underbrace{0.1}_{-_6} + \underbrace{0.1}_{-_7} + \underbrace{0.1}_{-_8} = 0.3$
- Classifier 3: $E_3 = \underbrace{0.1}_{+_1} + \underbrace{0.1}_{+_2} + \underbrace{0.1}_{-_9} = 0.3$
- Choose classifier 1: $\hat{h}_1(\mathbf{x}) = h_1(\mathbf{x}),\ \varepsilon_1 = E_1 = 0.3$
- $\alpha_1 = \frac{1}{2} \ln\left(\frac{1-\varepsilon_1}{\varepsilon_1}\right) = 0.4236$

Example

Round 1: $k = 1$, $\alpha_1 = 0.4236$

i	Class	$W_k(i)$	$W_k(i)e^{-\alpha_k y_i \hat{h}_k(\mathbf{x}_i)}$	$W_{k+1}(i) = W_k(i)e^{-\alpha_k y_i \hat{h}_k(\mathbf{x}_i)}/Z_k$
1	$+_1$	0.1	$0.1e^{-0.4236} = 0.0655$	0.0655/0.9166 = 0.0715
2	$+_2$	0.1	$0.1e^{-0.4236} = 0.0655$	0.0655/0.9166 = 0.0715
3	$+_3$	0.1	$0.1e^{0.4236} = 0.1527$	0.1527/0.9166 = 0.1666
4	$+_4$	0.1	$0.1e^{0.4236} = 0.1527$	0.1527/0.9166 = 0.1666
5	$+_5$	0.1	$0.1e^{0.4236} = 0.1527$	0.1527/0.9166 = 0.1666
6	$-_6$	0.1	$0.1e^{-0.4236} = 0.0655$	0.0655/0.9166 = 0.0715
7	$-_7$	0.1	$0.1e^{-0.4236} = 0.0655$	0.0655/0.9166 = 0.0715
8	$-_8$	0.1	$0.1e^{-0.4236} = 0.0655$	0.0655/0.9166 = 0.0715
9	$-_9$	0.1	$0.1e^{-0.4236} = 0.0655$	0.0655/0.9166 = 0.0715
10	$-_{10}$	0.1	$0.1e^{-0.4236} = 0.0655$	0.0655/0.9166 = 0.0715

$$Z_1 = \sum_{i=1}^{10} W_1(i)e^{-\alpha_1 y_i \hat{h}_1(\mathbf{x}_i)} = 0.0655 \times 7 + 0.1527 \times 3 = 0.9166$$

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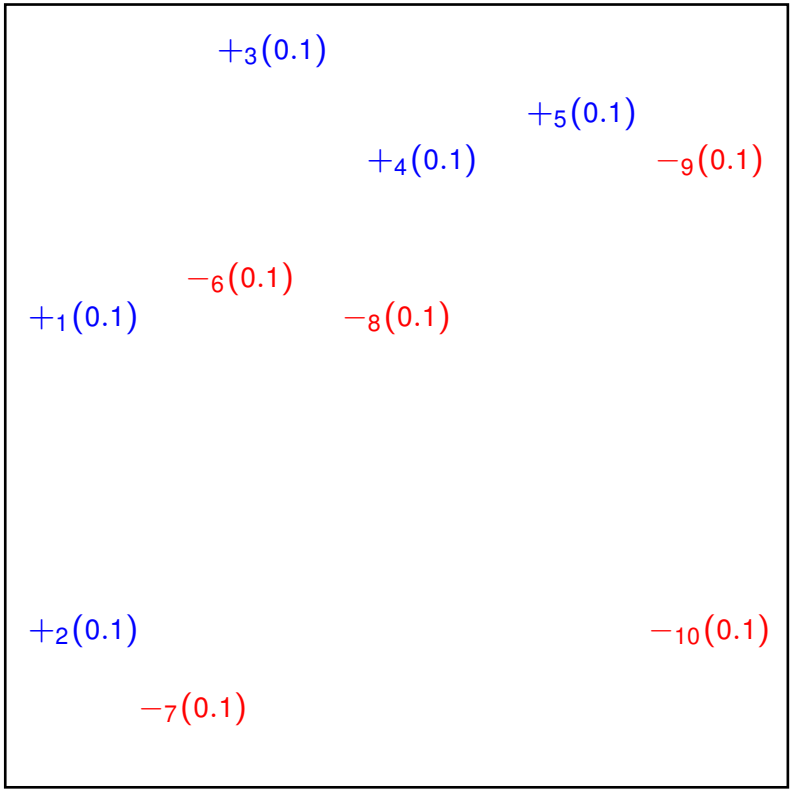
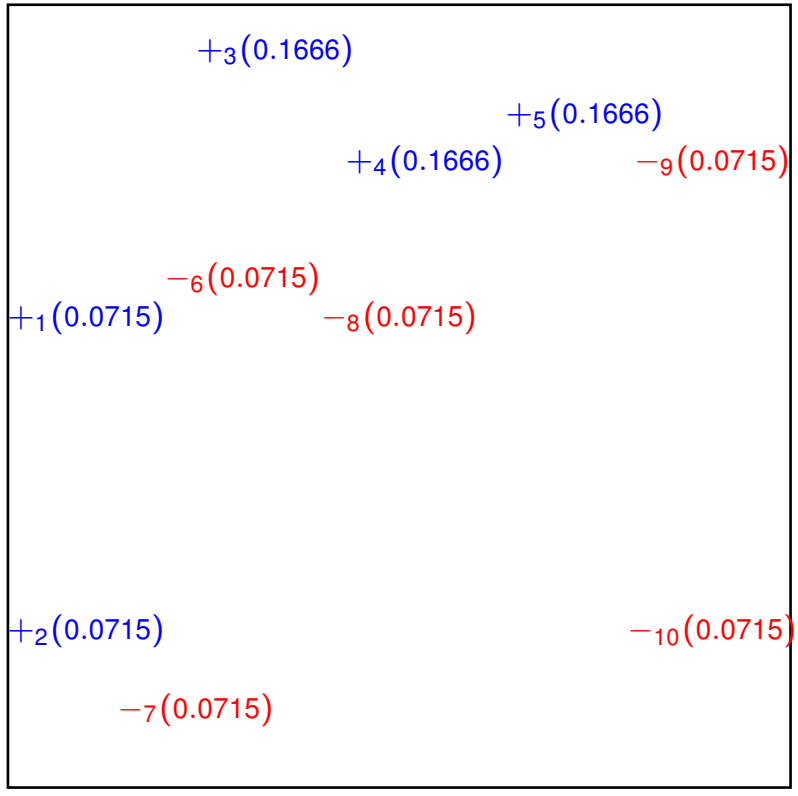
Example

In the beginning of round 1: $k = 1$



At the end of round 1: $k = 1$



Example

Round 2: $k = 2$

- Classifier 1: $E_1 = \underbrace{0.1666}_{+_3} + \underbrace{0.1666}_{+_4} + \underbrace{0.1666}_{+_5} = 0.4998$
- Classifier 2: $E_2 = \underbrace{0.0715}_{-_6} + \underbrace{0.0715}_{-_7} + \underbrace{0.0715}_{-_8} = 0.2145$
- Classifier 3: $E_3 = \underbrace{0.0715}_{+_1} + \underbrace{0.0715}_{+_2} + \underbrace{0.0715}_{-_9} = 0.2145$
- Choose classifier 2 (i.e., $\hat{h}_2(\mathbf{x}) = h_2(\mathbf{x})$) and $\varepsilon_2 = E_2 = 0.2145$
- $\alpha_2 = \frac{1}{2} \ln\left(\frac{1-\varepsilon_2}{\varepsilon_2}\right) = 0.6490$

Example

Round 2: $k = 2$, $\alpha_2 = 0.6490$

i	Class	$W_k(i)$	$W_k(i)e^{-\alpha_k y_i \hat{h}_k(\mathbf{x}_i)}$	$W_{k+1}(i) = W_k(i)e^{-\alpha_k y_i \hat{h}_k(\mathbf{x}_i)}/Z_k$
1	$+_1$	0.0715	$0.0715e^{-0.6490} = 0.0374$	0.0374/0.8213 = 0.0455
2	$+_2$	0.0715	$0.0715e^{-0.6490} = 0.0374$	0.0374/0.8213 = 0.0455
3	$+_3$	0.1666	$0.1666e^{-0.6490} = 0.0871$	0.0871/0.8213 = 0.1061
4	$+_4$	0.1666	$0.1666e^{-0.6490} = 0.0871$	0.0871/0.8213 = 0.1061
5	$+_5$	0.1666	$0.1666e^{-0.6490} = 0.0871$	0.0871/0.8213 = 0.1061
6	$-_6$	0.0715	$0.0715e^{0.6490} = 0.1368$	0.1368/0.8213 = 0.1666
7	$-_7$	0.0715	$0.0715e^{0.6490} = 0.1368$	0.1368/0.8213 = 0.1666
8	$-_8$	0.0715	$0.0715e^{0.6490} = 0.1368$	0.1368/0.8213 = 0.1666
9	$-_9$	0.0715	$0.0715e^{-0.6490} = 0.0374$	0.0374/0.8213 = 0.0455
10	$-_{10}$	0.0715	$0.0715e^{-0.6490} = 0.0374$	0.0374/0.8213 = 0.0455

$$Z_2 = \sum_{i=1}^{10} W_2(i)e^{-\alpha_2 y_i \hat{h}_2(\mathbf{x}_i)} = 0.0374 \times 4 + 0.0871 \times 3 + 0.1368 \times 3 = 0.8213$$

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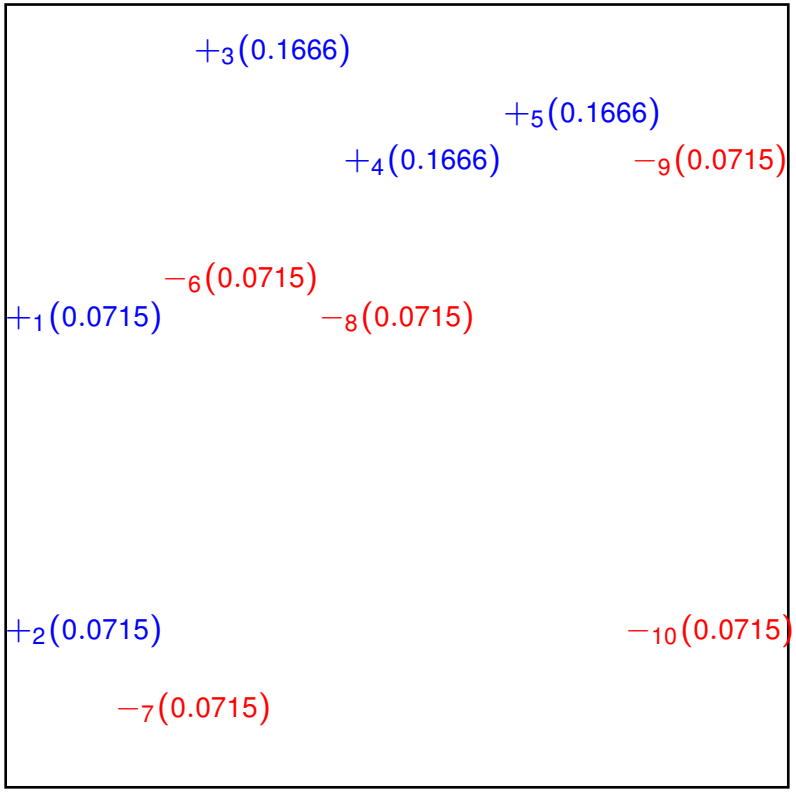
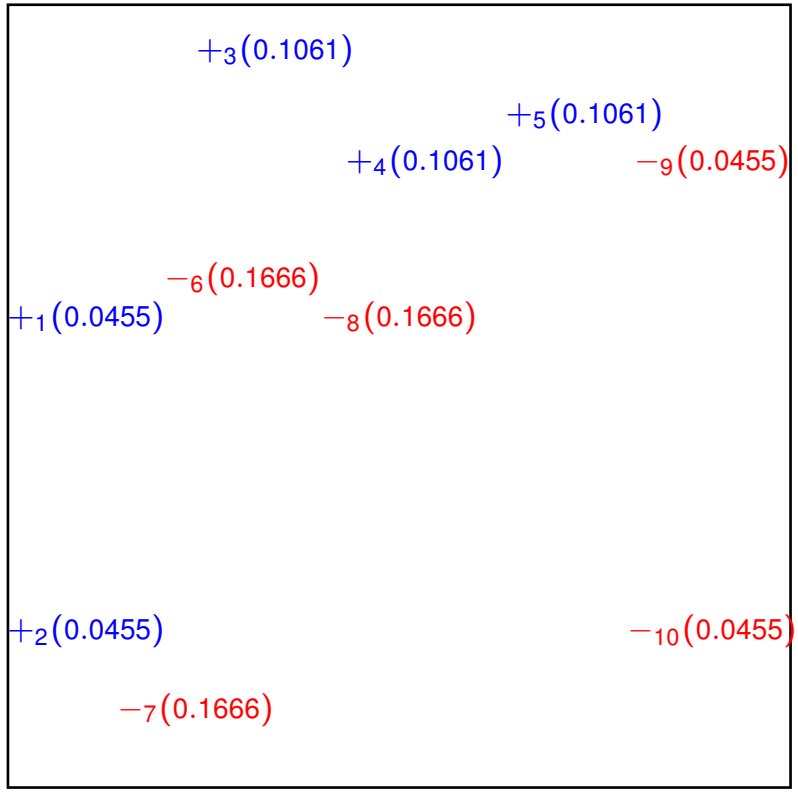
Example

In the beginning of round 2: $k = 2$



At the end of round 2: $k = 2$



Example

Round 3: $k = 3$

- Classifier 1: $E_1 = \underbrace{0.1061}_{+_3} + \underbrace{0.1061}_{+_4} + \underbrace{0.1061}_{+_5} = 0.3183$
- Classifier 2: $E_2 = \underbrace{0.1666}_{-_6} + \underbrace{0.1666}_{-_7} + \underbrace{0.1666}_{-_8} = 0.4998$
- Classifier 3: $E_3 = \underbrace{0.0455}_{+_1} + \underbrace{0.0455}_{+_2} + \underbrace{0.0455}_{-_9} = 0.1365$
- Choose classifier 3 (i.e, $\hat{h}_3 = h_3(\mathbf{x})$) and $\varepsilon_3 = E_3 = 0.1365$
- $\alpha_3 = \frac{1}{2} \ln\left(\frac{1-\varepsilon_3}{\varepsilon_2}\right) = 0.9223$

Example

Final classifier:

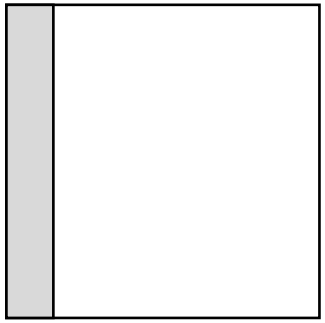
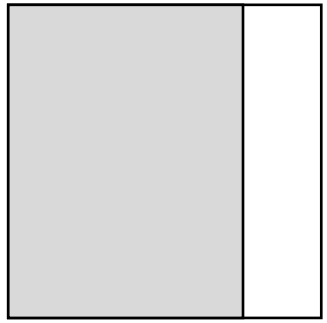
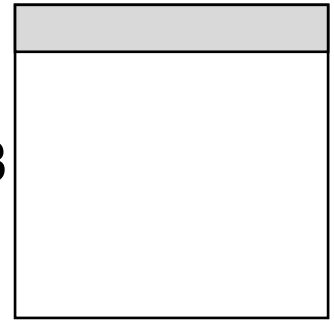
$$H(\mathbf{x}) = \text{sign}\left(\sum_{k=1}^{k_{\max}} \alpha_k h_k(\mathbf{x})\right)$$

$$= \text{sign}(0.4236 h_1(\mathbf{x}) + 0.6490 h_2(\mathbf{x}) + 0.9223 h_3(\mathbf{x}))$$

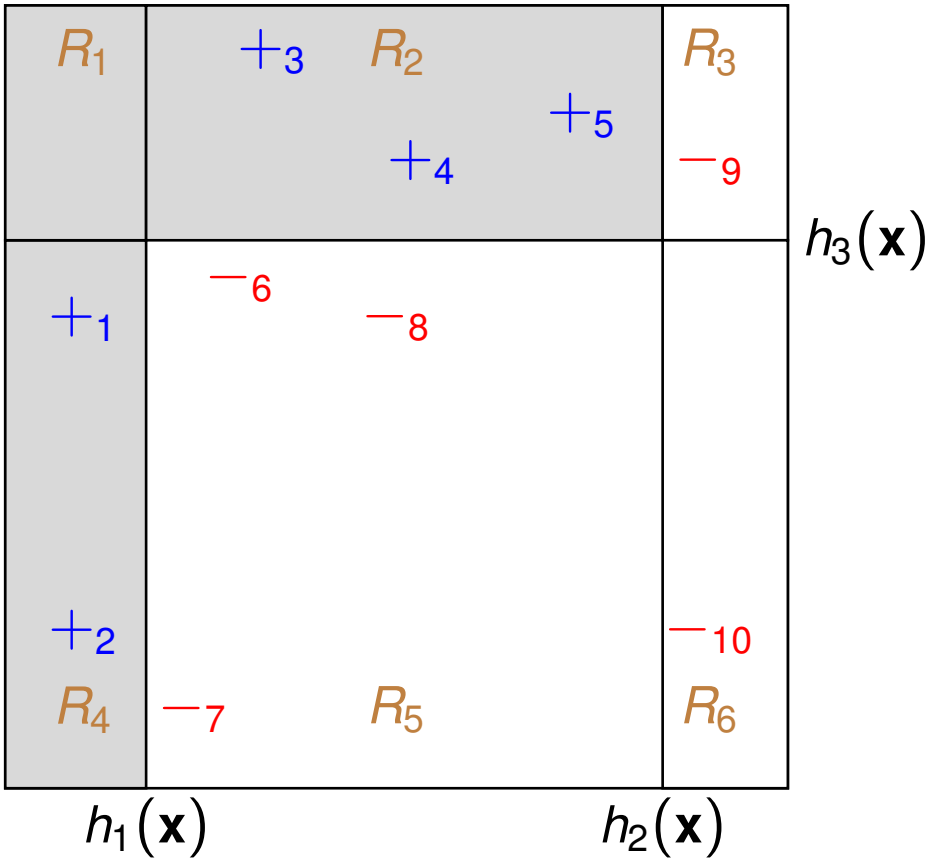
Example

Final classifier:

$H(\mathbf{x}) =$

sign(0.4236  $h_1(\mathbf{x})$ +0.6490  $h_2(\mathbf{x})$ +0.9223  $h_3(\mathbf{x})$)

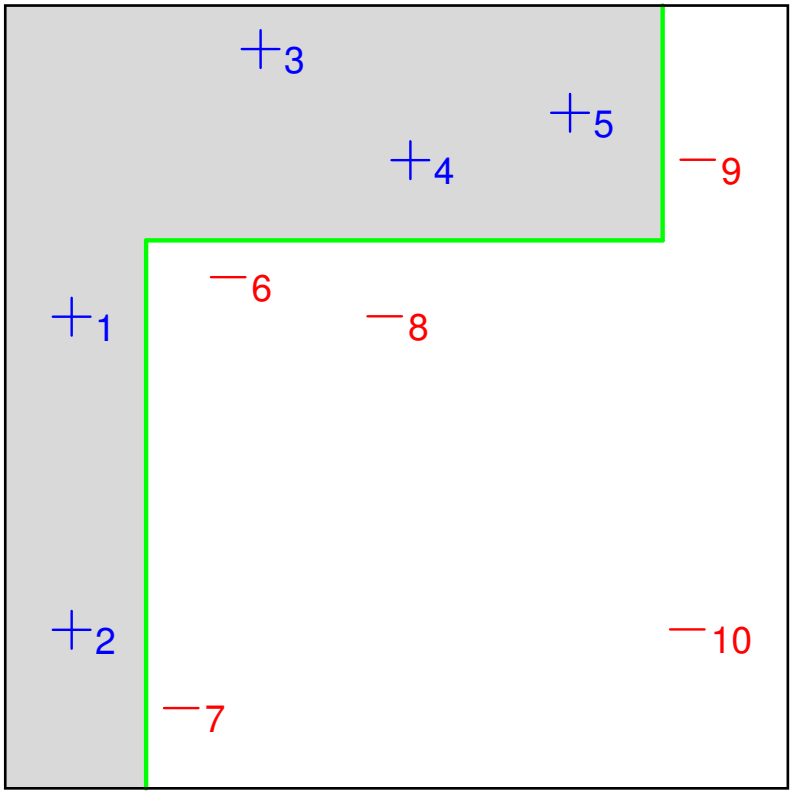
Example



- $0.4236h_1(\mathbf{x}) + 0.6490h_2(\mathbf{x}) + 0.9223h_3(\mathbf{x})$
- $R_1 : 0.4236 \times 1 + 0.6490 \times 1 + 0.9223 \times 1 = 1.9949$
- $R_2 : 0.4236 \times -1 + 0.6490 \times 1 + 0.9223 \times 1 = 1.1477$
- $R_3 : 0.4236 \times -1 + 0.6490 \times -1 + 0.9223 \times 1 = -0.1503$
- $R_4 : 0.4236 \times 1 + 0.6490 \times 1 + 0.9223 \times -1 = 0.1503$
- $R_5 : 0.4236 \times -1 + 0.6490 \times 1 + 0.9223 \times -1 = -0.6969$
- $R_6 : 0.4236 \times -1 + 0.6490 \times -1 + 0.9223 \times -1 = -1.9949$

Adaboost - Adaptive Boost

Example



- $0.4236h_1(\mathbf{x}) + 0.6490h_2(\mathbf{x}) + 0.9223h_3(\mathbf{x})$
- $R_1 : 0.4236 \times 1 + 0.6490 \times 1 + 0.9223 \times 1 = 1.9949$
- $R_2 : 0.4236 \times -1 + 0.6490 \times 1 + 0.9223 \times 1 = 1.1477$
- $R_3 : 0.4236 \times -1 + 0.6490 \times -1 + 0.9223 \times 1 = -0.1503$
- $R_4 : 0.4236 \times 1 + 0.6490 \times 1 + 0.9223 \times -1 = 0.1503$
- $R_5 : 0.4236 \times -1 + 0.6490 \times 1 + 0.9223 \times -1 = -0.6969$
- $R_6 : 0.4236 \times -1 + 0.6490 \times -1 + 0.9223 \times -1 = -1.9949$